GENERAL

- IDL is very handy at operating on arrays. Since a matrix is easily represented as an array, IDL has included many useful matrix operations.
- You can use these operations to do many interesting things, including image processing, finding the eigenvalues and eigenvectors of a matrix, and solving systems of linear equations.

MATRIX MULTIPLICATION

- The matrix multiplication operator is the double pound sign, “##”, which multiplies the rows of the first array with the columns of the second array.

```
a = lindgen(2,3)
b = lindgen(3,2)

print, a =>
0 1
2 3
4 5

print, b =>
0 1 2
3 4 5

print, a##b =>
3 4 5
15 24 33
```

- NOTE: The operator single pound sign operator “#” multiplies the columns of the first array with the rows of the second array, which is opposite of normal matrix multiplication. When doing matrix multiplication, **make sure you use the double pound sign!**
- Arrays can be transposed using the `transpose()` function.

```
a = lindgen(3,2)

print, a =>
0 1 2
3 4 5
```
print, transpose(a) => 1 4
2 5

DETERMINANTS AND TRACES OF MATRICES
• The `DETERM` and `TRACE` functions calculate the determinant and trace of a matrix.

\[
a = \begin{pmatrix}
3 & 5 & -2 \\
2 & 5 & 4 \\
-2 & -7 & 9
\end{pmatrix}
\]

print, determ(a) => -97.0
print, trace(a) => 17.0

FINDING THE INVERSE OF A MATRIX
• The `INVERT` function finds the inverse of a square matrix.

SYSTEMS OF LINEAR EQUATIONS
• The system of linear equations

\[
\begin{align*}
a_{00}x_0 + a_{10}x_1 + a_{20}x_2 &= b_0 \\
a_{01}x_0 + a_{11}x_1 + a_{21}x_2 &= b_1 \\
a_{02}x_0 + a_{12}x_1 + a_{22}x_2 &= b_2
\end{align*}
\]

can be written in matrix form as

\[
\begin{pmatrix}
a_{00} & a_{10} & a_{20} \\
a_{01} & a_{11} & a_{21} \\
a_{02} & a_{12} & a_{22}
\end{pmatrix}
\begin{pmatrix}
x_0 \\
x_1 \\
x_2
\end{pmatrix}
= 
\begin{pmatrix}
b_0 \\
b_1 \\
b_2
\end{pmatrix}
\]

which can also be written as

\[
Ax = b.
\]

• A is called the coefficient matrix.

• If the right-hand side is zero, the system of equations is homogeneous, and has non-trivial solutions only if the determinant of the coefficient matrix is singular (equal to zero).
• If the right-hand side is not zero, the system of equations is *nonhomogeneous*, and has a unique solution only if the determinant of the coefficient matrix is nonsingular (not equal to zero).

**Cramer’s Rule**

• Cramer’s rule states that the solutions to a system of $n$ equations is given by:

$$x_n = \frac{D_n}{D}$$

where $D$ is the determinant of the coefficient matrix $A$, and $D_n$ is the determinant of the matrix formed by replacing the $n$th column of $A$ with $b$.

  o Note: Cramer’s rule can only yield the trivial solution for homogenous equations, because $D_n$ would always be zero.

• Using the system of three equations above as an example, Cramer’s Rule would give the solutions as

$$x = \begin{pmatrix} \frac{D_0}{D} \\ \frac{D_1}{D} \\ \frac{D_2}{D} \end{pmatrix}$$

where

$$D = \begin{vmatrix} a_{00} & a_{10} & a_{20} \\ a_{01} & a_{11} & a_{21} \\ a_{02} & a_{12} & a_{22} \end{vmatrix}; \quad D_0 = \begin{vmatrix} b_0 & a_{10} & a_{20} \\ b_1 & a_{11} & a_{21} \\ b_2 & a_{12} & a_{22} \end{vmatrix}; \quad D_1 = \begin{vmatrix} a_{00} & b_0 & a_{20} \\ a_{01} & b_1 & a_{21} \\ a_{02} & b_2 & a_{22} \end{vmatrix}; \quad D_2 = \begin{vmatrix} a_{00} & a_{10} & b_0 \\ a_{01} & a_{11} & b_1 \\ a_{02} & a_{12} & b_2 \end{vmatrix}.$$

• IDL has a `cramer` function, which solves a system of $n$ linear equations using Cramer’s Rule.

  o By defining arrays for the coefficient matrix and the right-hand side of the matrix equation as

$$a = \begin{pmatrix} a_{00} & a_{10} & a_{20} \\ a_{01} & a_{11} & a_{21} \\ a_{02} & a_{12} & a_{22} \end{pmatrix}, \quad b = \begin{pmatrix} b_0 \\ b_1 \\ b_2 \end{pmatrix}$$

we can then use the `cramer` function.
\[ x = \text{CRAMER}(a, b) \]

and \( x \) will be an array containing the solution values for \( x_0, x_1, \) and \( x_2. \)

- The input arrays to the \texttt{CRAMER} function must be floating-point or double-precision.
- The coefficient matrix must be non-singular.

OTHER MATRIX/LINEAR ALGEBRA FUNCTIONS
- IDL has a host of other functions and procedures for manipulating matrices and solving systems of linear equations, and for solving eigenvalue problems. For a complete list, look up matrix, linear algebra, or linear equation in the online help.