EXERCISES

1. Show that if the wind is blowing parallel to the isotherms that the temperature advection is zero.

**Answer:** If wind is parallel to isobars, then $V$ and $\nabla T$ are normal to each other. Therefore, their dot product is zero.

6. **a.** For the following profile of $u$, explain whether a downdraft would cause an increase or decrease in $u$ at the location of the dot. Assume that $u$ is constant in $x$ and $y$ [$u = u(t, z)$]

Hint: \[
\frac{\partial u}{\partial t} = -\vec{V} \cdot \nabla u = -u \frac{\partial u}{\partial x} - v \frac{\partial u}{\partial y} - w \frac{\partial u}{\partial z}
\]

**Answer:** \[
\frac{\partial u}{\partial t} = -w \frac{\partial u}{\partial z}; \quad w < 0, \frac{\partial u}{\partial z} < 0 \quad \therefore \frac{\partial u}{\partial t} < 0
\]

**b.** Do the same as in 6.a., only for the following profile

**Answer:** \[
\frac{\partial u}{\partial t} = -w \frac{\partial u}{\partial z}; \quad w < 0, \frac{\partial u}{\partial z} > 0 \quad \therefore \frac{\partial u}{\partial t} > 0
\]
7. Prove the following identities:

<table>
<thead>
<tr>
<th></th>
<th>( \frac{\partial \hat{i}}{\partial x} = \tan \phi \frac{j}{a} - \frac{1}{a} \hat{k} )</th>
<th>( \frac{\partial \hat{j}}{\partial x} = -\tan \phi \hat{i} )</th>
<th>( \frac{\partial \hat{k}}{\partial x} = \frac{1}{a} \hat{i} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\partial \hat{i}}{\partial y} = 0 )</td>
<td>( \frac{\partial \hat{j}}{\partial y} = -\frac{1}{a} \hat{k} )</td>
<td>( \frac{\partial \hat{k}}{\partial y} = \frac{1}{a} j )</td>
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<tr>
<td>( \frac{\partial \hat{i}}{\partial z} = 0 )</td>
<td>( \frac{\partial \hat{j}}{\partial z} = 0 )</td>
<td>( \frac{\partial \hat{k}}{\partial z} = 0 )</td>
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**Answer for** \( \frac{\partial \hat{i}}{\partial x} \) and \( \frac{\partial \hat{j}}{\partial x} \), **which are the most difficult.**

\( \frac{\partial \hat{i}}{\partial x} \) : Refer to diagram below:

From the diagrams above we see that

\[
\left| \frac{\partial \hat{i}}{\partial x} \right| = \lim_{\delta x \to 0} \frac{\delta \hat{i}}{\delta x} = \lim_{\delta x \to 0} \frac{\hat{i}(x + \delta x) - \hat{i}(x)}{\delta x} = \lim_{\delta x \to 0} \frac{\theta}{\delta x}.
\]

The angle \( \theta \) is also equal to

\[
\theta = \frac{\delta x}{R} = \frac{\delta x}{a \cos \phi}
\]

so that

\[
\left| \frac{\partial \hat{i}}{\partial x} \right| = \lim_{\delta x \to 0} \frac{\theta}{\delta x} = \frac{1}{a \cos \phi}.
\]

The direction is oriented along the line perpendicular to the axis of rotation, so from the diagram on the left we see that

\[
\frac{\partial \hat{i}}{\partial x} = \frac{\partial \hat{i}}{\partial x} \left( \sin \phi \hat{j} - \cos \phi \hat{k} \right) = \frac{1}{a \cos \phi} \left( \sin \phi \hat{j} - \cos \phi \hat{k} \right) = \frac{\tan \phi}{a} \hat{j} - \frac{1}{a} \hat{k}
\]
\[
\frac{\partial \hat{j}}{\partial x} : \text{ Refer to diagram below:}
\]

From the diagrams above we see that

\[
\lim_{\delta x \to 0} \frac{\delta \hat{j}}{\delta x} = \lim_{\delta x \to 0} \left( \frac{\hat{j}(x + \delta x) - \hat{j}(x)}{\delta x} \right) = \lim_{\delta x \to 0} \frac{\theta}{\delta x}.
\]

The angle \( \theta \) is also equal to

\[
\theta = \frac{\delta x}{S} = \frac{\delta x}{a/\tan \phi}
\]

so that

\[
\frac{\partial \hat{j}}{\partial x} = \lim_{\delta x \to 0} \frac{\theta}{\delta x} = \tan \phi \cdot \frac{a}{\Delta x}.
\]

The direction is oriented westward, so that

\[
\frac{\partial \hat{j}}{\partial x} = -\tan \phi \cdot \frac{a}{\Delta x}.
\]