ESCI 241 – Meteorology
Lesson 12 – Geopotential, Thickness, and Thermal Wind
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GEOPOTENTIAL

- **The acceleration due to gravity is not constant. It varies from place to place, with the largest variation due to latitude.**
  - **What we call gravity is actually the combination of the *gravitational* acceleration and the centrifugal acceleration due to the rotation of the Earth.**
  - **Gravity at the North Pole is approximately 9.83 m/s^2, while at the Equator it is about 9.78 m/s^2.**
- **Though small, the variation in gravity must be accounted for. We do this via the concept of *geopotential*.**
- **A surface of constant geopotential represents a surface along which all objects of the same mass have the same potential energy (the potential energy is just $m\Phi$).**
- **If gravity were constant, a geopotential surface would also have the same altitude everywhere. Since gravity is not constant, a geopotential surface will have varying altitude.**
- ***Geopotential* is defined as**

$$\Phi \equiv \int_0^z g\,dz\,, \tag{1}$$

or in differential form as

$$d\Phi = g\,dz\,. \tag{2}$$

- ***Geopotential height* is defined as**

$$Z \equiv \frac{\Phi}{g_0} = \frac{1}{g_0}\int_0^z g\,dz \tag{3}$$

where g_0 is a constant called *standard gravity*, and has a value of 9.80665 m/s^2.

- **If the change in gravity with height is ignored, geopotential height and geometric height are related via**

$$Z = \frac{g}{g_0}z\,. \tag{4}$$

  - **If the local gravity is stronger than standard gravity, then $Z > z$.**
  - **If the local gravity is weaker than standard gravity, then $Z < z$.**

- **Gravity varies from around 9.79 to 9.82 m/s^2. Therefore, $g/g_0 \cong 1$, and for many applications we can ignore the difference between geopotential and geometric height, since $Z \cong z$.**
  - **But, keep in mind that they are different, and at times this difference, though small, is very important and cannot be neglected.**
- **The surface of a liquid at rest will lie along a surface of constant geopotential, or along a surface of constant geopotential height.**
- **<u>We can convert any equation from geometric height to geopotential height by simply substituting Z for z, and g_0 for g</u>.**

THE HYPSOMETRIC EQUATION

- **The hydrostatic equation in terms of geopotential height is**

$$\frac{dp}{dZ} = -\rho g_0 . \tag{5}$$

- **Substituting for density from the ideal gas law we have**

$$\frac{dp}{dZ} = -\frac{pg_0}{R_d T} ,$$

or

$$dZ = -\frac{R_d}{g_0}\frac{T}{P}dp . \tag{6}$$

- **Integrating (6) between two levels in the atmosphere gives**

$$\int_{Z_1}^{Z_2} dZ = -\frac{R_d}{g_0}\int_{p_1}^{p_2}\frac{T}{p}dp \tag{7}$$

or

$$Z_2 - Z_1 = -\frac{R_d}{g_0}\int_{p_1}^{p_2} T\frac{dp}{p} = \frac{R_d}{g_0}\int_{p_2}^{p_1} T\frac{dp}{p} . \tag{8}$$

  - **Using the generalized mean value theorem of calculus this becomes**

$$Z_2 - Z_1 = \frac{R_d}{g_0}\bar{T}\int_{p_2}^{p_1}\frac{dp}{p} \tag{9}$$

where

$$\bar{T} = \frac{1}{\ln\left(p_1/p_2\right)} \int_{p_2}^{p_1} T \frac{dp}{p} \quad \textbf{(10)}$$

is the average temperature in the layer between *p*$_1$ and *p*$_2$.

- **The formula for the geopotential distance between the two pressure levels is therefore**

$$Z_\Delta = Z_2 - Z_1 = \frac{R_d}{g_0} \bar{T} \ln\left(\frac{p_1}{p_2}\right) \quad \textbf{(11)}$$

and is called the *hypsometric equation*.

- **The hypsometric equation tells us that the *thickness* or difference in geopotential height (Z_Δ) between two pressure levels is proportional to the average temperature of the layer between the two levels.**
  - **A colder average temperature equals a smaller thickness.**
  - **A warmer average temperature equals a larger thickness.**
- **We often plot thickness instead of isotherms.**
- **For moist air we would simply substitute the layer-average virtual temperature for the temperature,**

$$\bar{T}_v = \frac{1}{\ln\left(p_1/p_2\right)} \int_{p_2}^{p_1} T_v \frac{dp}{p}. \quad \textbf{(12)}$$

THE THERMAL WIND

- **The *thermal wind* is defined as the vector difference in the geostrophic wind between two levels of the atmosphere**

$$\vec{V}_T \equiv \vec{V}_{g2} - \vec{V}_{g1}. \quad \textbf{(13)}$$

- **The thermal wind tells us how the geostrophic wind changes with height.**
- **The geostrophic wind in pressure coordinates is**

$$\vec{V}_g = \frac{g_0}{f} \hat{k} \times \nabla Z \quad \textbf{(14)}$$

Therefore, the thermal wind is

$$\vec{V}_T = \vec{V}_{g2} - \vec{V}_{g1} = \frac{g_0}{f} \hat{k} \times \nabla Z_2 - \frac{g_0}{f} \hat{k} \times \nabla Z_1 = \frac{g_0}{f} \hat{k} \times \nabla \left(Z_2 - Z_1\right).$$

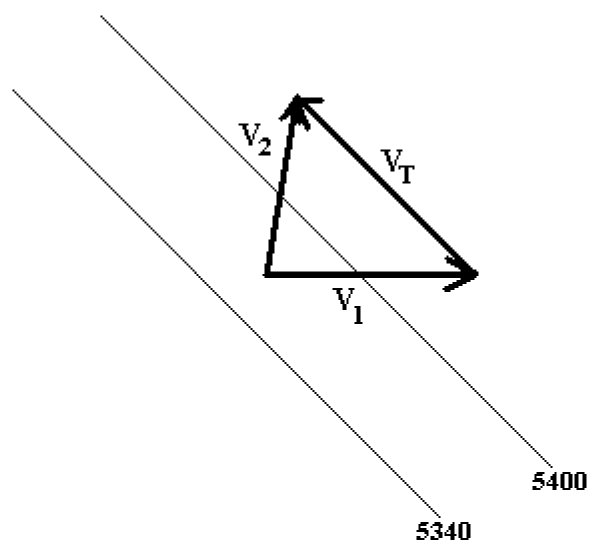
***Z*$_2$ – *Z*$_1$ is just the thickness, Δ*Z*, of the layer. Therefore, the thermal wind is**

$$\vec{V}_T = \frac{g_0}{f}\hat{k}\times\nabla Z_\Delta. \qquad \textbf{(15)}$$

- **This equations tells us that *the thermal wind will be oriented parallel to the thickness contours, with low thickness to the left*.**
- **Using the hypsometric equation (11) we can write the thermal wind equation (15) in terms of the layer-average temperature gradient as**

$$\vec{V}_T = \frac{R_d}{f}\ln\left(p_1/p_2\right)\hat{k}\times\nabla \bar{T}. \qquad \textbf{(16)}$$

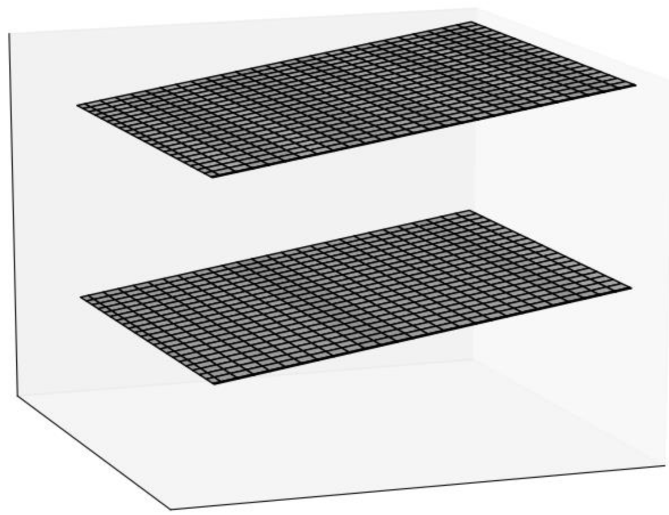
- **Since thickness is a measure of the average temperature of the layer, *the thermal wind will be oriented with lower temperatures to the left*.**
- **The equation for the thermal wind (15) looks nearly identical to the equation for the geostrophic wind, only with thickness (temperature) gradient instead of pressure (or height) gradient.**
- **The rules for the thermal wind are:**
  - **Blows parallel to the thickness lines with low temperatures to the left (Northern Hemisphere).**
  - **Tighter thickness gradient leads to stronger thermal wind.**
- **The thermal wind is useful because if we have a map of the thickness contours we can estimate the winds aloft from the winds at lower levels.**



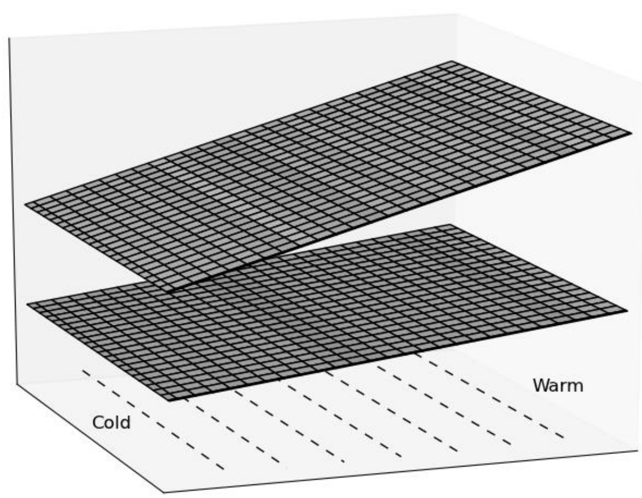
PHYSICAL EXPLANATION OF THERMAL WIND

- **The physical basis for the thermal wind can be explained as follows.**

- **First, remember that on a constant pressure surface the geostrophic wind is normal to the height gradient, and the speed is proportional to the slope of the pressure surface.**
- **Second, remember that the thickness between two pressure surfaces is proportional to the average temperature in the layer.**
- **If there is no thermal gradient in the layer, an upper level-pressure surface will be sloped the same as the lower-level pressure surface, and so the geostrophic wind on each surface will be identical.**



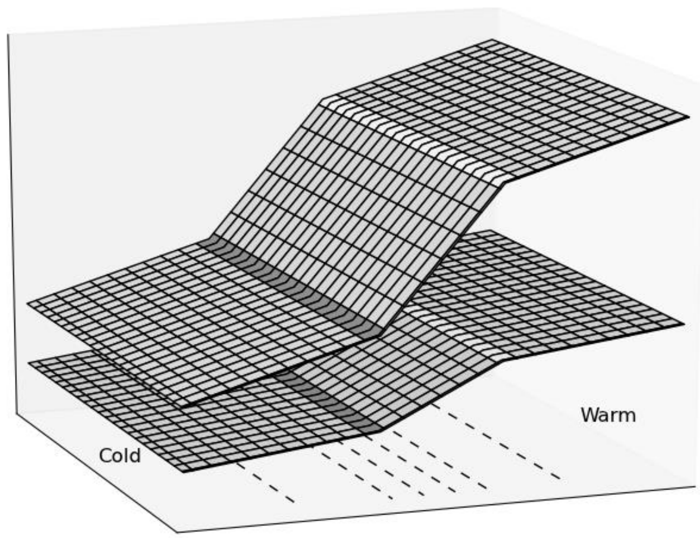
- **If there is a thermal gradient in the layer, the upper-level surface will have a different slope than the lower-level surface, and therefore a different geostrophic wind.**



THE THERMAL WIND EXPLAINS THE JET STREAM

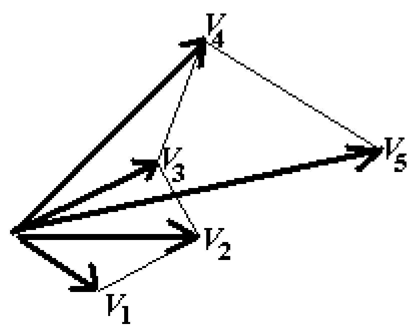
- **Thermal wind explains the existence of the jet stream.**

- **In the vicinity of a front, pressure surfaces are tilted more steeply, and the higher in the atmosphere the steeper the slope will be.**

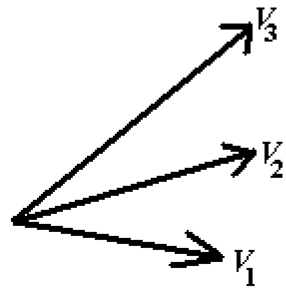


BACKING AND VEERING WINDS

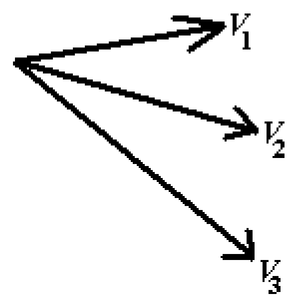
- **A *hodograph* is a graph made by placing the tails of the wind vectors at different levels together, and then drawing a line that sequentially connects their heads in ascending order (see example below)**



- ***Backing winds* are winds whose vectors rotate counter-clockwise (either with time or with height).**



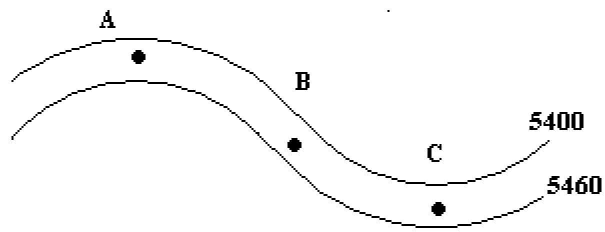
- ***Veering winds* are winds whose vectors rotate clockwise (either with time or with height).**



- **The thermal wind leads to the following relations between the winds on a hodograph and temperature advection.**
  - **Veering winds indicate warm-air advection**
  - **Backing winds indicate cold-air advection**

EXERCISES

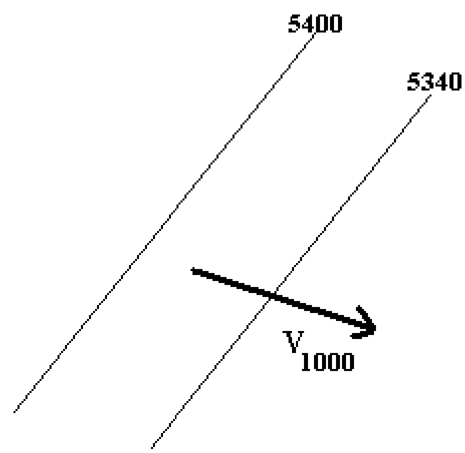
1. a. For the 500 mb isohypse pattern below calculate the magnitude of the geostrophic wind at point B. Assume a horizontal distance between isobars of 150 km, and $f = 10^{-4}\ s^{-1}$. The contour heights are given in meters.



b. For points A, B, and C tell whether you would expect the actual wind to be greater than, less than, or equal to the geostrophic wind.

2. Find the thickness of the 1000 mb to 500 mb layer for a layer average temperature of –7°C.

3. The diagram below shows the 1000 – 500 mb thickness contours and the 1000 mb wind.



a. Calculate the magnitude of the thermal wind. Use a horizontal distance between the thickness contours of 200 km and $f = 10^{-4}s^{-1}$.

b. Assuming that the 1000 mb wind is 15 m/s, draw the thermal wind on the diagram. Also draw the 500 mb wind and estimate its magnitude.

c. Will the 1000 – 500 mb layer be warming or cooling in this case?